

**Damjan PANTIĆ, Matthias DEES, Marko MARINKOVIĆ,  
Bojan TUBIĆ, Dragan BOROTA, Zoran TOMOVIĆ<sup>1</sup>**

## **DYNAMIC PROGRAMMING APPROACH TO ADDRESSING CHALLENGES IN FOREST MANAGEMENT**

### **SUMMARY**

This paper presents a model of dynamic programming in addressing specific challenges encountered in management planning of clonal poplar plantations. The planning challenge is posed by the need to make a decision that will result in maximum revenue, a possibility to select different rotations, plant spacing and the frequency of thinning during the rotations. The aim of this research was to analyze only options that are realistically possible, since the number of possible combinations is extremely high. The dynamic programming model is tested in the Republic of Serbia (Province of Vojvodina) on clonal poplar plantations in the forests managed by the Public Enterprise "Vojvodinašume". The analysis included possible thinning in the 10<sup>th</sup> or the 20<sup>th</sup> year (or both) and three possible rotations (20, 25 and 30 years), taking into account three types of plant spacing that are most commonly used in planting in alluvial region of Vojvodina, along the Danube River (6x3 m, 5x5 m and 6x6 m). The result of the analysis showed that the optimal procedure for the management of Euro-American poplar (*Populus-euroamericana* I - 214) stands, in the given conditions, in the long term, involves afforestation with plant spacing 5x5 m, thinning in the 10<sup>th</sup> year and the rotation length of 25 years.

**Keywords:** dynamic programming, forest management planning, optimization, maximization of revenue.

### **INTRODUCTION**

Forestry planning is a complex challenge because forests are complex ecological and economic systems with a number of factors affecting the process of planning and decision-making (Balteiro and Romero, 2008). Large systems, such as forests, face an issue of a viewpoint, i.e. an issue of a comprehensive analysis prior to making a decision. The practice proves that this process considers only some of the realities of such a system, while other aspects are seen less clearly (Medarević, 2006; Curovic et al., 2012; Pantić et al., 2013). Based on

---

<sup>1</sup> Damjan Pantić, Dragan Borota (corresponding author: dragan.borota@sfb.bg.ac.rs): University of Belgrade, Faculty of Forestry, Department of Forest Management Planning, Kneza Višeslava 1, 11000 Belgrade, SERBIA; Mathias Dees: University of Freiburg, Faculty of Environment and Natural Resources, Institute of Forest Sciences, Tennenbacher Straße 4, 79085 Freiburg, GERMANY; Marko Marinković, Bojan Tubić, Zoran Tomović: Public Enterprise Vojvodinašume, Preradovićevo 2, 21000 Novi Sad, SERBIA.

Note: The authors declare that they have no conflicts of interest. Authorship Form signed online.

this fact, every decision maker, involved in any form of forest planning, faces a number of criteria that influence his decision (Balteiro and Romero, 2008). Compared to other complex systems, the aforementioned problem is even more pronounced in forestry since modern forest management planning implies a need to explore complex and multifunctional goals that must be achieved within a specified period. This has been particularly evident in recent years when the economic orientation of the forest has been shifting towards ecology and ecological principles, where profit is no longer the only measure of sustainable management. Therefore, the optimization of decision making and planning in forestry should be formulated within the framework of decisions based on multiple criteria (MCDM – Multiple criteria decision-making paradigm) (Balteiro and Romero, 2008).

Multiple-criteria decision-making methods are applied in cases when a decision making process requires considering different options, where the choice thereof is affected by hardly comparable criteria and conflicting interests. Many authors have analyzed application of multi-criteria decision making in forest management planning (Mendoza and Prabhu, 2000; Bousson, 2001; de Steiguer *et al.*, 2002; Kangas *et al.*, 2005; Sheppard *et al.*, 2005; Šporčić *et al.*, 2010). Their papers describe standard decision models and their practical application in forest management planning. In fact, these models are based on the principles of optimization, where forest management strategies are identified, evaluated and selected based on their suitability for achieving objectives.

One of the widespread methods in this field is mathematical programming. Mathematical programming is the generally accepted term for a set of methods that can be used in the forest management planning in order to optimize the goals and overcome the constraints imposed by various influences (constraints due to specific forestry production) and conflicting interests (Vladimirov and Chudnenko 2009; Bettinger *et al.*, 2009). Within the mathematical programming, a dynamic programming is one of the most important methods. It is important to note that the term "programming", which is used in this context, is not directly related to computer programming, but also came into wide use in decision theory.

Dynamic programming is perhaps the most widely used stand-level optimization process except linear programming. A development of linear programming method in the forestry has begun 60 years ago (Curtis, 1962; Bell and Enoch, 1977; Weintraub *et al.*, 2000; Bettinger *et al.*, 2009). From the 1970s through 1990s, significant advances occurred in the application of dynamic programming to forest management problems. Early researches mostly emphasized economic or commodity production goals. However, significant work continues today exploring the use of the approach to recognize and accommodate environmental and social objectives. Dynamic programming facilitates the examination of a large number of alternatives for the management of a stand of trees by reducing the range of options explored (Bettinger *et al.*, 2009).

Dynamic programming has often been used to maximize biological potential (annual increment) and economic returns generated by a stand of trees and represent a technique for the systematic determination of optimal combinations of decisions. This is also a method for numerically solving a dynamic system of equations. The range of types of problems that can be solved is extremely wide and encompasses not only business and industrial applications, but many natural resource management issues as well (Kennedy, 1986). Since the management of forest and the resulting growth response of forests are both sequences of actions that may follow similar pathways at various points in time, sorting through a group of alternatives and selecting the optimum course of action may require multiple passes through the same data (Curovic et al., 2013). The dynamic programming approach gives opportunity to work iteratively through the sequence of decisions in a normal forward flowing fashion, or work backward through the sequence from the ending condition to the beginning, without having to make the same calculation twice (Bettinger et al., 2009).

The subject and purpose of the research were defined in accordance with above considerations. This paper focuses on dynamic programming with a view of exploring the possibility of its application in forest management planning of Euro-American poplar forests in the Public Enterprise "Vojvodinasume". The purpose of the research was to implement this method in the decision-making process in forestry so as to ensure and facilitate such process. The planning issues is posed by the need to make a decision that will result in maximum revenue, a possibility to select different rotations, plant spacing and the frequency of thinning during the rotations of poplar forests. Since the number of possible combinations is extremely high, even hundreds, the analysis presented in this paper are limited to the possible combinations which represent realistic examples of forest management.

## MATERIAL AND METHODS

This paper focuses on determining the maximum revenue using dynamic programming method. For this purpose, three types of plant spacing which are most commonly used in planting in the alluvial region of Vojvodina, along the Danube River, are taken into consideration. These specific types of plant spacing are 6x3 m, 5x5 m and 6x6 m. The paper also considers three possible rotations of 20, 25 and 30 years, and possible thinning in the 10<sup>th</sup> or the 20<sup>th</sup> year (or both). Each of the above planting distances implies various costs since planting costs depend on the number of holes that are drilled, and the number of seedlings. The costing method which is normally used by PE "Vojvodinasume" was applied here as well and included the costs of preparing the soil for planting and costs of silvicultural measures. The analysis is limited to a specific number of options just to provide a balance between illustrating technical details of applied method and providing a realistic management example. The total area of poplar plantations of clone I-214, that was used to explore the optimal management procedure, covered more than 10,000 ha. Yields per hectare in some ages were taken as the

average of the realized annual cut in the PE “Vojvodinasume” for Euro-American poplar clone I-214 from 2009 – 2014. Given a set of yields for the thinnings and final harvests, and assumptions regarding the wood prices for the mixture of products (which varies by age given assumptions of mixture of pulpwood, fuel wood and saw timber volume), the "rewards" associated with each branch in the dynamic programming network were developed.

$$r(xx, a \rightarrow b) \quad (1)$$

The “reward” for going from node  $a$  to node  $b$  on a branch in the dynamic programming network, with an associated final harvest age of  $xx$ . The rotation length is open-ended, and  $r(a \rightarrow b)$  value is used to reflect their contribution to the soil expectation value. The contributions associated with branch in a dynamic programming network are compounded to the end of the rotation, and then discounted to the present to reflect incurring the cost or revenue perpetually in future rotations of the same set of treatments.

Each  $r(a \rightarrow b)$  value that is not associated with a final harvest is adjusted using the equation:

$$r^*(a \rightarrow b) = r(a \rightarrow b) \left[ \frac{(1+i)^{R-t}}{(1+i)^R - 1} \right] \quad (2)$$

where, the rotation age  $R$  is assumed in the analysis, with  $t$  = the time period in which the revenue is incurred.

Final harvest values are adjusted using the equation:

$$r^*(a \rightarrow b) = r(a \rightarrow b) \left[ \frac{1}{(1+i)^R - 1} \right] \quad (3)$$

Discounting of all the values at a particular moment provided comparable income streams and expenses that mature at different times. The method of discounting is a dynamic method of investment calculations, which shows cash flow models that mature at different times during the period of use of an investment property. This allows for achieving more realistic assessment of measures. The interest rate applied in this case is the discount rate of 5%.

## RESULTS

The network setup is illustrated in Figure 1, with the stages containing vertical columns of nodes. Every stage requires decision-making process. In the case of stand – level management, this involves selecting a management action (including no actions at all). The numbers of steps (stages) reflect the stages needed to present adequately the reasonable options for managing sub-compartments. Stages of dynamic programming are the points of a problem with a number of different states of such problem. The states imply possible problem-related choices that can be made within individual steps (nodes at each stage). In other words, states are various possible stand structural conditions that might exist if a certain course of action is chosen. Each stage has a specific number of states (Figure 1).

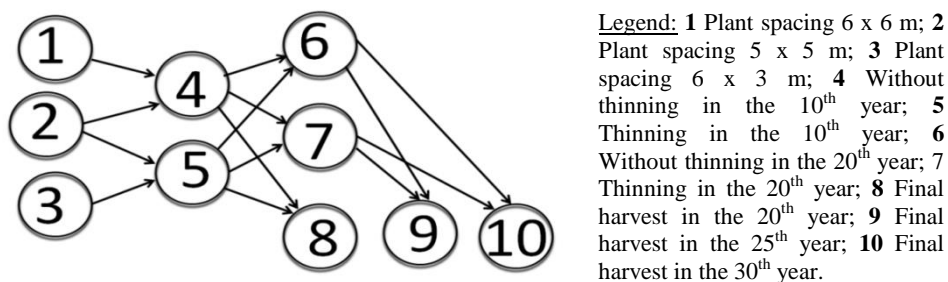


Figure 1: A network of the possible choices related to the presented forest management problem - Stage 1; Stage 2 - (Age 10); Stage 3 - (Age 20); (Age 25); (Age 30)

Discounted revenues and costs associated with different management alternatives are calculated, or in other words, the “reward” for going from node a to node b on a branch in the dynamic programming network (Table 1).

Table1. Discounted revenues and costs associated with different management alternatives

From node	To node	Volume harvested (m <sup>3</sup> ·ha <sup>-1</sup> )	Price (EUR)	Final harvest (age)	Revenue/cost (EUR)	Discount factor	r(xx, a→b) (EUR)
1	4			20	-1,315.79	1.604852	-2,111.65
1	4			25	-1,315.79	1.419049	-1,867.17
1	4			30	-1,315.79	1.301029	-1,711.88
2	4			20	-1,578.95	1.604852	-2,533.98
2	4			25	-1,578.95	1.419049	-2,240.60
2	4			30	-1,578.95	1.301029	-2,054.26
2	5	30.00	15.79	20	473.68	0.985240	-2,067.08
2	5	30.00	15.79	25	473.68	0.871173	-1,827.94
2	5	30.00	15.79	30	473.68	0.798719	-1,675.92
3	5	30.00	15.79	20	473.68	0.985240	-2,348.84
3	5	30.00	15.79	25	473.68	0.871173	-2,076.90
3	5	30.00	15.79	30	473.68	0.798719	-1,904.17
4	6		0.00	25	0.00	0.00	0.00
4	6		0.00	30	0.00	0.00	0.00
4	7	50.00	21.93	25	1,096.49	0.534825	586.43
4	7	50.00	21.93	30	1,096.49	0.490344	537.66
4	8	300.00	21.93	20	6,578.95	0.604852	3,979.29
5	6			25	0.00	0.00	0.00
5	6			30	0.00	0.00	0.00
5	7	25.00	26.32	25	657.89	0.534825	351.86
5	7	25.00	26.32	30	657.89	0.490344	322.59
5	8	300.00	26.32	20	7,894.74	0.604852	4,775.15
6	9	430.00	30.70	25	13,201.75	0.419049	5,532.18
6	10	540.00	29.82	30	16,105.26	0.301029	4,848.15
7	9	400.00	30.70	25	12,280.70	0.419049	5,146.22
7	10	500.00	29.82	30	14,912.28	0.301029	4,489.02

For example, moving from node 1 to node 4 is associated only with the cost of site preparation and planting.

$$r(20, 1 \rightarrow 4) = 1315.79 \left[ \frac{(1,05)^{20-0}}{1,05^{20} - 1} \right] = 2111.65 \text{ EUR} \quad (4)$$

On the other side, moving from node 2 to node 5 is associated with the combination of costs (of site preparation and planting) and the revenue (potential thinning value).

$$1578.95 \left[ \frac{(1,05)^{20-0}}{1,05^{20} - 1} \right] = 2533.98 \text{ EUR} \quad 473.68 \left[ \frac{(1,05)^{20-10}}{1,05^{20} - 1} \right] = 466.9 \text{ EUR} \quad (5)$$

$$r(20, 2 \rightarrow 5) = 466.9 - 2533.98 = 2067.08 \text{ EUR} \quad (6)$$

The rewards  $r(xx, a \rightarrow b)$  for going from node  $a$  to node  $b$  with associated final harvest age of  $xx$  have been calculated in the same manner for other branches in the dynamic programming network.

Alternatives for this management issue are assessed using reverse method of dynamic programming. Backward recursion is advantageous for solving problems that contain options with the same time horizon. The issue will be addressed starting from the last stage, backward to the first stage. A decision must be made at each stage of the analysis that is guided by the notion that the best action for the stand will be chosen. Decisions involve transforming a state associated with one stage to a state associated with the next stage. Nodes reflect the entire set of decisions across the time span, and can be represented by a value that has been accumulated from the first/initial stage of the problem, or vice versa. Branches are the transitions that are possible from nodes at one stage to nodes at the next stage. A value is assigned to each branch indicating the benefit or cost associated with the transition from one state to another.

Having the above in mind, the analysis can begin at stage 3 by assessment of all the opportunities that result in the states described at stage 3 (Table 2).

Table 2. Analyzing all opportunities within the stage 3

	From node	Net revenue (EUR)	To node	Route
Stage 3	6	5,532.18	9	6-9
	6	4,848.15	10	6-10
	7	5,146.22	9	7-9
	7	4,489.02	10	7-10

As a result of this analysis:

$R_6 = 5,532.18 \text{ EUR}$

$R_7 = 5,146.22 \text{ EUR}$

In Table 2, “From node” means the node where a branch starts, while “to node” means the node where the branch ends. “The net revenue” means the accumulated revenue associated with the route, and “the route” is the path

through the network represented by the revenue/cost.  $R_b$  is the maximum reward possible for moving along a particular path to node b. It equals the maximum value of  $R_a + r(a \rightarrow b)$  for all nodes that lead to node b.

As shown by the 20<sup>th</sup> year, when thinning is not performed, the most cost-effective option is clear cutting in the 25<sup>th</sup> year. In case of late thinning, cutting of the forest in the 25<sup>th</sup> year is a better option, rather than postponing it for the 30<sup>th</sup> year. Moving backward one stage, the result will be as follows (Table 3):

Table 3. Analyzing all opportunities within stage 2

	From node	Net revenue (EUR)	To node	Route
Stage 2	4	5,532.18	6	4-6-9
	4	5,732.65	7	4-7-9
	4	3,979.29	8	4-8
	5	5,532.18	6	5-6-9
	5	5,498.08	7	5-7-9
	5	4,775.15	8	5-8

As a result of this analysis:

$R_4=5,732.65$  EUR

$R_5=5,532.18$  EUR

In the 10<sup>th</sup> year, without thinning during this period, we can select one of the three options: no spacing until the end of the rotation; carry out the spacing around the 20<sup>th</sup> year; or perform clear-cutting (final harvest) in the 20<sup>th</sup> year. In this case, the best financial results are provided by the option that implies no thinning until the end of the rotation (where the rotation is a period of more than 20 years).

In the second case, when thinning is performed in the 10<sup>th</sup> year, we also have three choices: to manage forest stand after thinning with a rotation period of 20 years; no additional work, or to perform another thinning about 20<sup>th</sup> year and perform the final cutting in the 20<sup>th</sup> year. The best option in this case as well is to continue management after the 20<sup>th</sup> year, with no additional thinning. Finally, the previously mentioned stages included one more stage (Table 4).

Table 4. Analyzing all opportunities within stage 1

	From node	Net revenue (EUR)	To node	Route
Stage 1	2	3,492.04	4	2-4-7-9
	2	3,704.24	5	2-5-6-9
	3	3,455.28	5	3-5-6-9

As a result of this analysis:

$R_1=3,492.04$  EUR

$R_2=3,704.24$  EUR

$R_3=3,455.28$  EUR

The concept underlying the dynamic programming approach assumes that, regardless of earlier decisions, the remaining decisions will always constitute an optimal management regime, regardless of the state or intermediate stage. This principle is often referred to as the Principle of Optimality.

As can be seen from the analysis, the best option is to carry out afforestation with spacing 5x5 m, and then plan thinning around the 10<sup>th</sup> year. The 1-4-7-9 option was not considered because, in practice, thinning is not performed in stands of 6x6 m plant spacing since thinning would lead to insufficient number of trees in a forest. On the other hand, stands with 6x3 m plant spacing require mandatory thinning in the 10<sup>th</sup> year, which offers only one option that is feasible (3-5). Consequently, the optimal procedure for the management of Euro-American poplar stands, in the given circumstances, in the long term, involves the following:

1. Afforestation with plant spacing of 5x5 m;
2. Thinning in the 10<sup>th</sup> year;
3. Final harvest (rotation length) in the 25<sup>th</sup> year.

An interesting fact is a possibility to work iteratively through the sequence of decisions in a normal forward – flowing fashion or, as it is shown in this case, work backward through the sequence from the ending condition to the beginning condition without having to make the same calculation twice.

## DISCUSSION

As already pointed out, the issue of the application of multiple-criteria decision making in forest management planning was researched by numerous authors who described the standard decision making models and their practical application in forest management planning (Mendoza and Prabhu, 2000; Bousson, 2001; de Steigueretal, 2002; Kangas et al., 2005; Sheppard et al., 2005; Šporčić et al., 2010). Weintraub gave an example of application of linear programming through application of mixed integer programming (Weintraub et al., 2000). Principles of optimization were used, where forest management strategies were identified, evaluated and selected based on their suitability to achieve objectives.

In forestry practice, many decisions are taken at different times and at different hierarchical levels of management. The decisions which are made during the planning are often exposed to pressures and effects of numerous factors. In these cases, when several elements affecting the decision need to be evaluated, it is necessary to introduce systems that will facilitate assessment of the criteria for identifying the optimal solution and thus lead to the optimization of the process, or the optimal result (Balteiro and Romero, 2008). Different rotations, plant spacing and the frequency of thinning during rotations are often the subject of discussion of scientific and professional audiences in Vojvodina. Using dynamic programming is an attempt to facilitate a decision making process that is not biased.



Dynamic programming has often been used to maximize biological potential (annual increment) and an economic return generated by a stand of trees, and represents a technique for a systematic determination of optimal combinations of decisions (Kennedy, 1986). Zadnik (1996) has applied dynamic programming approach to determine the sequence of decisions required to take the forest from its existing state towards the target state, i.e., a state in which economics, ecology and social goals are achieved at a maximum “possible and desired” level. He also used the Bellman’s principle of optimality and recursive method to identify an optimal sequence of alternatives from the set of sequences when the decision makers’ relative preferences on the objectives are known. In our study, management has to make a decision on rotations, planting distances and the sequence thinning during rotations, and be able to justify the choices. This method can be applied in this situation and could lead to an optimal solution (Zadnik, 2004).

The use of the coppice regeneration method for the definition of both the optimal harvest age in each cycle and the optimal number of coppice cycles within a full rotation was presented in 2012. That paper developed a stochastic dynamic programming approach to development of the management alternatives (e.g., fuel treatment, stool thinning, coppice cycles, and rotation length) that maximizes expected net revenues (Ferreira, 2012). In view of the fact that forest industry often has no time for experiments that last for many years, dynamic programming could be used as an “effective tool”, by the set of linear equations defining the goals and constraints (criteria) that are automated considered on the basis of given variables, and thus leads to the optimal solution for the problem given (Bettinger et al., 2009).

The dynamic programming approach can certainly be used as a solution for sequencing problems. It can be employed in a number of important situations and, when applicable, it offers many advantages. Poor decisions are usually blamed on the analytical methodology; however, it is the interpretation of the results by the manager and the judgment of the manager that matters in the end (Williams, 1988). Ability to assess a number of alternatives for the management of trees and stands is a fundamental step in the management of forests, and demonstrates your competence as a manager of resources. Clearly, implementation of this methodology can be very important for the practical solution of management problems encountered by forestry enterprises in Serbia. Optimum solutions for minor problems can be identified by only a small and precisely predictable amount of computation. Larger problems lend themselves to a successive – approximation technique which, although lacking rigorous justification, has worked remarkably well in practice. Finally, these techniques can be mechanized completely by rather simple computer software. The planning challenge addressed by this paper imposed by a need to make a decision that will result in maximum revenues. The possible decisions relate to several possible rotations, planting distances and the frequency of thinning during rotations in the poplar plantations managed by the Public Enterprise “Vojvodinašume”.

Asante and his associates presented results of a dynamic programming model used to determine the optimal harvesting decision for a forest stand in the boreal forest of western Canada that provides both timber harvest volume and carbon sequestration services. The results of that study indicated that optimal harvesting age is relatively insensitive to carbon stocks in dead organic matter (Asante et al., 2011).

## CONCLUSION

The analysis of the data presented in this paper leads to the conclusion that dynamic programming method can be successfully used in addressing management issues when different rotations, plant spacing and the frequency of thinning planting have to be selected, if the same set of treatments repeats perpetually in future rotations. We have demonstrated applicability of the method to a realistic management issue. Ultimately, selection of a preferred management alternative using dynamic programming method is highly influenced by the quality of the used data. The model allows the analysis of different management alternatives and it represents a tool for studying the generated effects of the management. In the future work, the model structure needs to be analyzed further and applied to other poplar clones and other sites.

The testing and application of this methodology in the forestry of Serbia is of high importance, because these methods are not developed well in the forestry science, and they are completely unknown in practice. This paper takes a very simple example in order to illustrate, as simply as possible, a possibility of applying dynamic programming in our conditions. Relevant scientific literature and the results of this study indicate that there are realistic grounds for practical application of this method in addressing complex issues of planning and management in forestry. Advantages of this method are particularly noticeable in multiple-criteria decision-making or in circumstances influenced by numerous relevant factors and criteria.

## ACKNOWLEDGEMENTS

This paper is supported by the Public Enterprise “Vojvodinašume” and the Ministry of Education, Science and Technological Development of the Republic of Serbia under projects TR 37008

## REFERENCES

- Asante, P., G.W. Armstrong, and W.L. Adamowicz (2011): Carbon sequestration and the optimal forest harvest decision: A dynamic programming approach considering biomass and dead organic matter. *Journal of Forest Economics* 17: 3-17.
- Balteiro, L.D., and C. Romero (2008). Making forestry decisions with multiple criteria: A review and an assessment. *Forest Ecology and Management* 255: 3222–3241.
- Bettinger, P., K. Boston, J.P. Siry and D.L. Grebner (2009): *Forest Management and Planning*. Elsevier 30 Corporate Drive, Suite 400, Burlington, MA 01803 USA, pp 163-181.

- Bell, E.F. (1977): Mathematical programming in forestry. *Journal of Forestry* 75: 317-319.
- Bousson, E. (2001): Development of a multicriteria decision support system adapted to multiple-use forest management: application to forest management at the management unit level in Southern Belgium. In: Franc, A., Laroussinie, O. and Karjalainen, T. (eds.). *Criteria and indicators for sustainable forest management at the forest management unit level*. Joensuu: European Forest Institute. *EFI Proceedings* 38: 151-164.
- Curtis, F.H. (1962): Linear programming the management of a forest property. *Journal of Forestry* 60: 611-616.
- Curovic, M. and Spalevic V. (2012): Development of spruce trees in mixed forests of spruce, fir and beech (*Piceeto-Abieti-Fagetum*) on Mt. Ljubišnja. *Agriculture & Forestry* 53: 11-18.
- Curovic, M., Spalević V. and Medarević M., (2013): The ratio between the real and theoretically normal number of trees in mixed fir, beech and spruce forests in the National park "Biogradska gora". *Agriculture & Forestry* Vol.59 issue 1, p: 7-17.
- De Steiguer, J.E., L. Liberti, A. Schuler, and B. Hansen (2002): *Multi-Criteria Decision Models for Forestry and Natural Resources Management. An Annotated Bibliography*, United States Department of Agriculture, Forest Service, Northeastern Research Station, NE-307.
- Ferreira, L., M.F. Constantino, J.G. Borges and J. Garcia-Gonzalo (2012): A Stochastic Dynamic Programming Approach to Optimize Short-Rotation Coppice Systems Management Scheduling: An Application to Eucalypt Plantations under Wildfire Risk in Portugal. *Forest Science* 58: 353-365.
- Kangas, J., R. Store, and A. Kangas (2005): Socioecological landscape planning approach and multicriteria acceptability analysis in multiple-purpose forest management. *Forest Policy and Economics* 7: 603-614.
- Kennedy, J.O.S. (1986): *Dynamic Programming. Applications to Agriculture and Natural Resources*. Elsevier Applied Science Publishers, London, 341 p.
- Medarević, M. (2006): Strategic planning, In *Forest Management Planning*, University of Belgrade, Faculty of Forestry, Belgrade, pp 113-135.
- Mendoza, G.A. and R. Prabhu (2000): Multiple criteria decision-making approaches to assessing forest sustainability using criteria and indicators: a case study. *Forest Ecology and Management* 174: 329-343.
- Pantić, D., N. Stojković, D. Borota, B. Tubić and M. Marinković (2013): Possibilities of applying sampling measurement in the inventory of structurally heterogeneous stands. *Agriculture & Forestry* 59: 65-79.
- Sheppard, S.R.J. and M. Meitner (2005): Using multi-criteria analysis and visualization for sustainable forest management planning with stakeholder groups. *Forest Ecology and Management* 207: 171-187.
- Šporčić, M., M. Landekić, M. Lovrić, S. Bogdan and K. Šegotić (2010): Multi-criteria decision making in support of forest management - models and experiences. *Forestry Society* 5-6: 275-286.
- Vladimirov, I.N. and A.K. Chudnenko (2009): Multilevel Modeling of the Forest Resource Dynamics. *Mathematical Modeling of Natural Phenomena* 4: 72-88.
- Weintraub, A., R.L. Church, A.T. Murray and M. Guignard (2000): Forest management models and combinatorial algorithms: analysis of state of the art. *Annals of Operations Research* 96: 271-285.

- Williams, MRW. (1988): Decision-Making in Forest Management. 2nded Research Studies Press, Ltd. Letchworth, Hertwordshire, England, 133 p.
- Zadnik, S.L. (1996): Interactive, dynamic and multi-attribute technique for modeling in forest planning. Publication No. FWS 1-96, Virginia Polytechnic Institute and State.
- Zadnik S.L. (2004): Integrating the fuzzy analytic hierarchy process with dynamic approach for determining the optimal forest management decisions. Ecological modeling, 194: 296-305